ELECTRIC FIELD PROFILE IN MULTILAYER STRUCTURES

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Chapter 1

Introduction

Surface plasmons are free metal electrons collective oscillations strongly confined to the surface and coupled to a propagate and vanishes electromagnetic wave [1]. Generally, characterization of this kind of resonances are obtained on reflectometry experiments in conditions of total internal reflexion where is satisfied the condition of coupling of a vanishing electromagnetic wave with the surface plasmons. In metal dielectric multilayer systems is possible to obtain different plasmon modes in each interface that can not be directly identified from reflectometry experiments where the obtained measurements constitute the effective answer of the structure. Thus, it is necessary a complementary analysis which can be based on some model of optical response material, or more generally in an analytic calculation of the wave electromagnetic propagation in this kind of structures.

As the optic effective answer of this kind of structures depends strongly on the excitation of this kind of electromagnetic modes, this analysis would allow for the parameters and components structure appropriate to design structures with an advisable optic effective answer. In other words, it would allow us to design new materials with nonexistent naturally optical properties, [2, 3, 4] which ends improving the performance of current optical and optoelectronic devices, as well as in the creation of new technological possibilities.

In this work we present a calculation of electric field profile for a multilayer system metal-dielectric of n-layers embedded between two semi- infinitive dielectric medium. The calculation consist in solving the Maxwell's equations for a propagating wave. The results allows us also to describe the reflectance and transmitance functions of the structure.

1.1 Overall Objective

To develop a computing tool to calculate the electric field profile in stratified systems as function of the transverse coordinate to the structure.

1.2 Specific Objectives

- To review the calculation methods of the Fresnel's coefficients and the amplitudes of the electric field.

- To review the matrix transfer method for the calculation of the amplitudes of the reflected and transmitted waves in a multi-layer system.
- To implement the matrix transfer method in Python programming language.
- To develop computing code to calculate the distribution of the electric field across a system of n layers of arbitrary materials and thicknesses.

1.3 Summary of results and assessment of fulfillment of goals

The main result of this work is the computing code written in Python capable to calculate the distribution of the electric field along the transverse coordinate of a stratified system of n layers of arbitrary linear isotropic materials and arbitrary thicknesses, which is illuminated by an incident wave of arbitrary characteristics (linear polarization state, wavelength, and incidence angle). For brevity, in what follows we call *E-field profile* to the electric field distribution along the transverse coordinate to the system. The program includes the database of refractive indexes for most of the known materials as a function of the wavelength [5], which facilitates the automatic computation of the E-field profile for an arbitrary wavelength of the incoming electromagnetic wave.

On the other hand, although it was no planned in the design of the present work, we include a secondary code to compute the reflectance and transmitance functions of the multilayers. This code is a subroutine of the main program needed to test the main program. Since the reflectance and transmitance functions are well known for many configurations, and there are experimental results to contrast the calculations as well, this part of the program was fundamental in the validation of the calculations of the E-field profile. Many examples are also illustrated to verify the optimum performance of the code.

Another value added to this work is the fact of having written the whole code instead of using existing libraries already available for the transfer matrix method [5]. At first instance, we expected to use those libraries in order to simplify and optimize the present work, but due to difficulties to modify them, we finally decided to write the whole code. Accordingly, all the goals drawn for this work were reached beyond of the expectations.

1.4 Context and impact of the results

The results of the present work are an important component in the research currently developed in the Optics of Material Group (GOMa, by its acronyms in Spanish) lead by professor César Aurelio Herreño Fierro, who investigates on the plasmonics modes in mutilayer systems. His research is funded by the Centro de Investigaciones y Desarrollo Científico (CIDC) of the Universidad Distrital Francisco José de Caldas. In summary, this work is a computing program tool to study the behavior of the electric field inside stratified structures, from which the study of the optical resonances inside of such a structures, and its effect on the optical effective response of the structures, is possible.

1.5 Document structure

The rest of the document is organized as follows: In Chap. 2 we illustrate the details of the theoretical frame of the electrodynamics and optics of this work. In Chap. 3 we present the calculations of the E-field profile for the case of a single homogeneous isotropic layer by two different methods. Then, we extend the matrix method for the case of multilayer systems in Chap. 4. We finally present the details of the computational code in Chap. 5. Conclusions and perspectives are reserved for Chap. 6.

Chapter 2

Electromagnetic waves in matter

2.1 Fresnel Equations and Fresnel Coefficients

This work is concerned the behaviour of light in interaction with matter from the point of view of light as an electromagnetic wave. Therefore, the starting point of this document are the Maxwell's equations, of which the differential form in absence of free sources (charges and currents) is given by:

$$\nabla \cdot \mathbb{E} = 0 \tag{2.1}$$

$$\nabla \cdot \mathbb{B} = 0 \tag{2.2}$$

$$\nabla \times \mathbb{E} = -\frac{\partial \mathbb{B}}{\partial t} \tag{2.3}$$

$$\nabla \times \mathbb{H} = \frac{\partial \mathbb{D}}{\partial t} \tag{2.4}$$

Where $\mathbb{E}$ is the electric field intensity, $\mathbb{D}$ is the electric displacement, $\mathbb{B}$ is the magnetic induction and $\mathbb{H}$ is the magnetic field. [6]

2.1.1 Linear optical response

At first sight, the effect of electromagnetic radiation on matter can be describe by the dielectric and magnetic response of the material in terms of the dielectric polarization ($\mathbb{P}$) and the magnetization ($\mathbb{M}$), respectively. In linear media, i.e. for low intensity of the electric field, the dielectric polarization ($\mathbb{P}$) linearly depends on the electric field. It is to say

$$\mathbb{P} = \chi_e \epsilon_0 \mathbb{E}, \tag{2.5}$$

where ϵ_0 is the permitivity of the free space, and χ_e is the electric susceptibility. In the case of dielectric materials, χ_e measures how easy is to polarize the molecules of the material. It means, how easy is to induce an electric dipole into the molecules of the material.[7]

Similarly, the magnetization ($\mathbb{M}$) on such materials depends linearly on the magnetic field as follows

$$\mathbb{M} = \chi_m \mathbb{H}, \tag{2.6}$$

where χ_m is the magnetic susceptibility of the linear medium, and measures how easy is to induce a magnetic moment into the molecules of the material.

In this case, the dielectric displacement and the magnetic field in the material are related to the electric field and the magnetic induction, respectively, as follows

$$\mathbb{D} = \epsilon \mathbb{E} \tag{2.7}$$
$$\mathbb{H} = \frac{\mathbb{B}}{\mu}, \tag{2.8}$$

where ϵ and μ are the permitivity and permeability of the medium, respectively. Therefore, Maxwell's equations in linear media can be written as

$$\nabla \cdot \mathbb{E} = 0 \tag{2.9}$$
$$\nabla \cdot \mathbb{B} = 0 \tag{2.10}$$
$$\nabla \times \mathbb{E} = -\frac{\partial \mathbb{B}}{\partial t} \tag{2.11}$$
$$\nabla \times \mathbb{B} = \mu\epsilon \frac{\partial \mathbb{E}}{\partial t}, \tag{2.12}$$

where ϵ and μ describe the electric and magnetic response of the material trough their dependence on χ_e and χ_m, respectively. It is to say

$$\epsilon = \epsilon_0(1 + \chi_e) \tag{2.13}$$
$$\mu = \mu_0(1 + \chi_m). \tag{2.14}$$

2.1.2 Electromagnetic Waves

Eq. 2.9 to 2.12 are a coupled set of first order linear partial differential equations of $\mathbb{E}$ and $\mathbb{B}$. In uncoupling this system, a two independent second order linear differential equations result. These identical equations (one for $\mathbb{E}$ and one for $\mathbb{B}$) relate the second partial spatial derivative of the fields to its second partial time derivative, as follows

$$\nabla^2 \mathbb{E} = \epsilon\mu \frac{\partial^2 \mathbb{E}}{\partial t^2} \quad \text{and} \quad \nabla^2 \mathbb{B} = \epsilon\mu \frac{\partial^2 \mathbb{B}}{\partial t^2}. \tag{2.15}$$

These wave equations derived from Maxwell's equations are indicative of the wavy character of the electromagnetic fields in matter.[6] That is to say, the electric and magnetic fields obey wave equations and propagate as waves at the speed

$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{c}{n}, \tag{2.16}$$

where

$$n = \sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}} \tag{2.17}$$

is the refractive index of the material. Since for electromagnetic waves (EMW) in matter the amplitudes of the electric field is approximately c times the amplitude of the magnetic field (roughly speaking, a difference of eight orders of magnitude), all the optics is related to the electric field. It is to say that in optics the permeabilities of the materials are close to μ_0. In that sense ($\mu \approx \mu_0$), $n \approx \sqrt{\epsilon/\epsilon_0} = \sqrt{\epsilon_r}$, where $\epsilon_r = \epsilon/\epsilon_0$ is the relative permitivity of the medium.

Other important characteristics concerning the propagation of EMW in matter are derived from the fulfillment of the Maxwell's equations (Eq. 2.9 to Eq.2.12) by the wave function. That is, they are transverse waves in the sense that the electromagnetic fields do not have components along the propagation direction (in fulfillment of Eq. 2.9 and 2.10). In not-absorbing materials (ideal dielectrics) the electric and magnetic field of an EMW propagate in phase.

Monochromatic plane waves

The simplest solution to Eq. 2.1.2 is given for the case of the monochromatic plane wave. This kind of waves are mathematically easy to describe and all the important concepts concerning this work can be broadly analyzed on the basis of this kind of waves. For that we consider an electromagnetic monochromatic plane wave of angular frequency ω, wave vector $\vec{k}$, and amplitude E_0, described by

$$\mathbb{E} = \mathbb{E}_0 \exp i(\omega t - \vec{k} \cdot \vec{r}), \tag{2.18}$$

where $\vec{r}$ is the spatial coordinate. We are concerned with linear polarized waves. The polarization is given by the orientation of $\mathbb{E}_0$. Similar function is defined for the magnetic field of the EMW.[7]

2.1.3 Boundary conditions

Besides the propagating problem of electromagnetic waves in matter, this work mainly concerns with the effects undergone when an EMW goes throw a discontinuity in the propagating media. It is well known that when an EMW encounters a interface between two media, a fraction of the energy associate to the incident wave is reflected from the interface, and the rest of the energy is transmitted throw the interface, given place to a reflected and transmitted waves, respectively. In solving the problem of the relative amplitudes of the reflected and transmitted waves to the incident wave, we are concerned with a boundary conditions problem. The relation of the reflected and transmitted amplitudes to the incident amplitude is governed by the Maxwell's equations.

Let us start considering the problem of reflection and refraction for monochromatic plane waves at a plane boundary between two homogeneous isotropic media 1 and 2. The incoming wave meets the interface from the left side in medium 1, giving rise to a reflected wave back to medium 1, and a transmitted wave to medium 2. The general boundary conditions read:

$$\begin{aligned}
\epsilon_1 E_1^{\perp} &= \epsilon_2 E_2^{\perp} && (2.19)\\
B_1^{\perp} &= B_2^{\perp} && (2.20)\\
\mathbb{E}_1^{\|} &= \mathbb{E}_2^{\|} && (2.21)\\
\frac{1}{\mu_1}\mathbb{B}_1^{\|} &= \frac{1}{\mu_2}\mathbb{B}_2^{\|}. && (2.22)
\end{aligned}$$

Here, symbols $\perp$ and $\|$ denote, respectively, the perpendicular and parallel components of the field to the interface, and the subscript denotes the propagation media. In this case, fields with subscript 1 correspond to the superposition of the incoming (E_i) and reflected waves (E_r), while fields with subscript 2 correspond to the transmitted wave (E_t). Then, the boundary conditions can be rewritten as

$$\begin{aligned}
\epsilon_1(\mathbb{E}_i + \mathbb{E}_r)^{\perp} &= \epsilon_2 \mathbb{E}_t^{\perp} && (2.23)\\
(\mathbb{B}_i + \mathbb{B}_r)^{\perp} &= \mathbb{B}_t^{\perp} && (2.24)\\
\epsilon_1(\mathbb{E}_i + \mathbb{E}_r) &= \epsilon_2 \mathbb{E}_t^{\|} && (2.25)\\
\frac{1}{\mu_1}(\mathbb{B}_i + \mathbb{B}_r)^{\|} &= \frac{1}{\mu_2}\mathbb{B}_t^{\|}, && (2.26)
\end{aligned}$$

being ϵ_1 and ϵ_2 the permittivity constant of the respective media.

2.1.4 Reflection and transmission of P Waves (TM Wave)

P waves are also known as transverse magnetic (TM) waves because the magnetic field $\mathbb{H}$ is perpendicular to the plane of incidence.

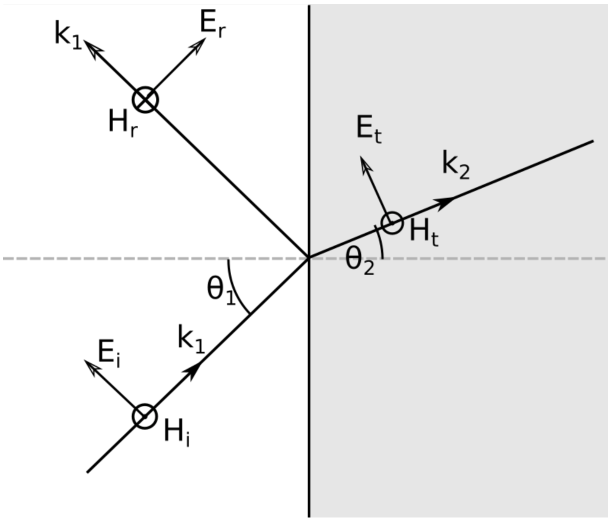


Figure 2.1: Reflection and transmission for P waves

Referring to Fig. 2.1, we consider the reflection and refraction of P waves. All electric field vectors are in the plane of incidence, and the magnetic field vectors are perpendicular to the plane of incidence, giving a positive energy flow in the direction of propagation.

In this geometry, the four boundary conditions read

$$\epsilon_1[-\mathbb{E}_i \sin\theta_i + \mathbb{E}_r \sin\theta_i] = -\epsilon_2\mathbb{E}_t \sin\theta_t \tag{2.27}$$

$$0 = 0 \tag{2.28}$$

$$\mathbb{E}_i \cos\theta_i + \mathbb{E}_r \cos\theta_i = \mathbb{E}_t \cos\theta_t \tag{2.29}$$

$$\sqrt{\frac{\epsilon_1}{\mu_1}}[\mathbb{E}_i - \mathbb{E}_r] = \sqrt{\frac{\epsilon_2}{\mu_2}}\mathbb{E}_t \tag{2.30}$$

Using the Snell Law $\frac{n_1}{n_2} = \frac{\sin\theta_t}{\sin\theta_i}$ in rewriting the Eq. 2.23, it gives

$$\begin{aligned}
\epsilon_1 \sin\theta_i[\mathbb{E}_i - \mathbb{E}_r] &= \epsilon_2\mathbb{E}_t \sin\theta_t \\
\epsilon_1[\mathbb{E}_i - \mathbb{E}_r] &= \epsilon_2\mathbb{E}_t \frac{\sin\theta_t}{\sin\theta_i} \\
\mathbb{E}_i - \mathbb{E}_r &= \mathbb{E}_t \frac{\epsilon_2}{\epsilon_1}\frac{n_1}{n_2}
\end{aligned}$$

Defining $\beta = \frac{\epsilon_2}{\epsilon_1}\frac{n_1}{n_2}$, we finally obtain

$$\mathbb{E}_i - \mathbb{E}_r = \beta\mathbb{E}_t \tag{2.31}$$

Following the same procedure for the Eq. 2.25, we have that

$$\begin{aligned}\cos\theta_i[\mathbb{E}_i + \mathbb{E}_r] &= \mathbb{E}_t \cos\theta_t \\ \mathbb{E}_i + \mathbb{E}_r &= \mathbb{E}_t \frac{\cos\theta_t}{\cos\theta_i}\end{aligned}$$

If $\alpha = \frac{\cos\theta_t}{\cos\theta_i}$, we get

$$\mathbb{E}_i + \mathbb{E}_r = \alpha \mathbb{E}_t \tag{2.32}$$

By adding Eq. 2.27 and 2.28, get:

$$\mathbb{E}_t = \frac{2}{\alpha+\beta}\mathbb{E}_i \tag{2.33}$$

$$\frac{\mathbb{E}_t}{\mathbb{E}_i} = \frac{2}{\alpha+\beta}. \tag{2.34}$$

Replacing Eq. 2.29 in Eq. 2.27

$$\frac{\mathbb{E}_r}{\mathbb{E}_i} = \frac{\alpha-\beta}{\alpha+\beta} \tag{2.35}$$

Knowing that the reflection and transmission coefficients are given by $r_p = \frac{\mathbb{E}_r}{\mathbb{E}_i}$ and $t_p = \frac{\mathbb{E}_t}{\mathbb{E}_i}$ respectively, and replacing α and β, we obtain

$$\begin{aligned} r_p &= \frac{\alpha-\beta}{\alpha+\beta} \\ r_p &= \frac{\frac{\cos\theta_t}{\cos\theta_i} - \frac{\epsilon_2}{\epsilon_1}\frac{n_1}{n_2}}{\frac{\cos\theta_t}{\cos\theta_i} + \frac{\epsilon_2}{\epsilon_1}\frac{n_1}{n_2}} \\ r_p &= \frac{\epsilon_1 n_2 \cos\theta_t - \epsilon_2 n_1 \cos\theta_i}{\epsilon_1 n_2 \cos\theta_t + \epsilon_2 n_1 \cos\theta_i}\end{aligned}$$

writing ϵ_1 in ϵ_2 terms and simplifying, we finally obtain the reflection coefficient given by

$$r_p = \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i} \tag{2.36}$$

For t_p we use the Eq. 2.30

$$\begin{aligned} r_p &= \frac{2}{\alpha+\beta} \\ r_p &= \frac{2}{\frac{\cos\theta_t}{\cos\theta_i} + \frac{\epsilon_2}{\epsilon_1}\frac{n_1}{n_2}} \\ r_p &= \frac{2\epsilon_1 n_2 \cos\theta_i}{\epsilon_1 n_2 \cos\theta_t + \epsilon_2 n_1 \cos\theta_i}\end{aligned}$$

writing ϵ_2 in ϵ_1 terms, and simplifying, we finally obtain the transmission coefficient given by

$$t_p = \frac{2 n_1 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i} \tag{2.37}$$

2.1.5 Reflection and transmission of S Waves (TE Wave)

S waves are also known as the transverse electric (TE) waves because the electric field $\mathbb{E}$ is transverse to the plane of incidence. Referring to Fig. 2.2, we consider the reflection and refraction for S waves. All electric field vectors are perpendicular to the plane of incidence, and the magnetic field vectors are chosen to give a positive energy flow in the direction of the wave vectors.

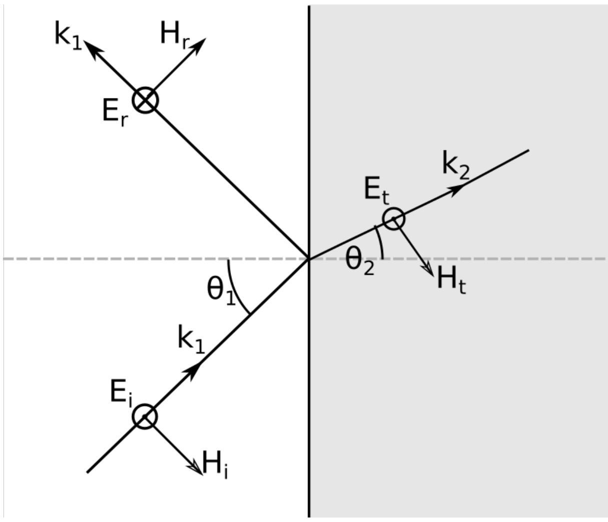


Figure 2.2: Reflection and transmission for S waves

Beginning from applying the four boundary conditions:

$$0 = 0 \quad (2.38)$$

$$\mathbb{B}_i \sin\theta_i + \mathbb{B}_r \sin\theta_i = \mathbb{B}_t \sin\theta_t \quad (2.39)$$

$$\mathbb{E}_i + \mathbb{E}_r = \mathbb{E}_t \quad (2.40)$$

$$\frac{1}{\mu_1}[-\mathbb{B}_i \cos\theta_i + \mathbb{B}_r \cos\theta_i] = -\frac{1}{\mu_2}\mathbb{B}_t \cos\theta_t, \quad (2.41)$$

and rewriting them in terms of $\mathbb{E}$, we obtain:

$$\sqrt{\frac{\epsilon_1}{\mu_1}}[\mathbb{E}_i \sin\theta_i + \mathbb{E}_r \sin\theta_i] = -\sqrt{\frac{\epsilon_2}{\mu_2}}\mathbb{E}_t \sin\theta_t \quad (2.42)$$

$$\mathbb{E}_i + \mathbb{E}_r = \mathbb{E}_t \quad (2.43)$$

$$\sqrt{\frac{\epsilon_1}{\mu_1}}[\mathbb{E}_i \cos\theta_i - \mathbb{E}_r \cos\theta_i] = \sqrt{\frac{\epsilon_2}{\mu_2}}\mathbb{E}_t \cos\theta_t \quad (2.44)$$

Then, the Eq. 2.40, reads

$$\sqrt{\frac{\epsilon_1}{\mu_1}}[\mathbb{E}_i - \mathbb{E}_r] = \sqrt{\frac{\epsilon_2}{\mu_2}}\mathbb{E}_t \frac{\cos\theta_t}{\cos\theta_i}$$

Then, in terms of α and β:

$$\mathbb{E}_i - \mathbb{E}_r = \alpha\beta\mathbb{E}_t \quad (2.45)$$

Adding the Eq. 2.39 and Eq.2.41 we get:

$$\mathbb{E}_t = \frac{2}{1+\alpha\beta}\mathbb{E}_i \quad (2.46)$$

$$\frac{\mathbb{E}_t}{\mathbb{E}_i} = \frac{2}{1+\alpha\beta} \quad (2.47)$$

Replacing Eq. 2.42 in Eq. 2.39:

$$\mathbb{E}_r = \frac{1-\alpha\beta}{1+\alpha\beta}\mathbb{E}_i \quad (2.48)$$

$$\frac{\mathbb{E}_r}{\mathbb{E}_i} = \frac{1-\alpha\beta}{1+\alpha\beta} \quad (2.49)$$

Following the same procedure for P waves, we finally obtain the reflection and transmission coefficients for S waves

$$r_s = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t} \quad (2.50)$$

$$t_s = \frac{2n_1\cos\theta_i}{n_1\cos\theta_i + n_2\cos\theta_t} \quad (2.51)$$

2.2 Dynamical Matrix

From the Eq. 2.25 and Eq. 2.26 where we evaluate the boundary conditions for P polarized waves:

$$\mathbb{E}_i\cos\theta_i + \mathbb{E}_r\cos\theta_i = \mathbb{E}_t\cos\theta_t \quad (2.52)$$

$$\sqrt{\frac{\epsilon_1}{\mu_1}}[\mathbb{E}_i - \mathbb{E}_r] = \sqrt{\frac{\epsilon_2}{\mu_2}}\mathbb{E}_t \quad (2.53)$$

Now, for symmetry we assume an artificial incoming wave from medium 2 to medium 1, identified as reflected primed amplitudes, as shown in Fig. (2.3). Thereafter, we will drop the artificial wave for the substrate, but for the moment we will keep it since this approach leads us to the generalized matrix method for solving the problem of propagation of electromagnetic waves in layered media.

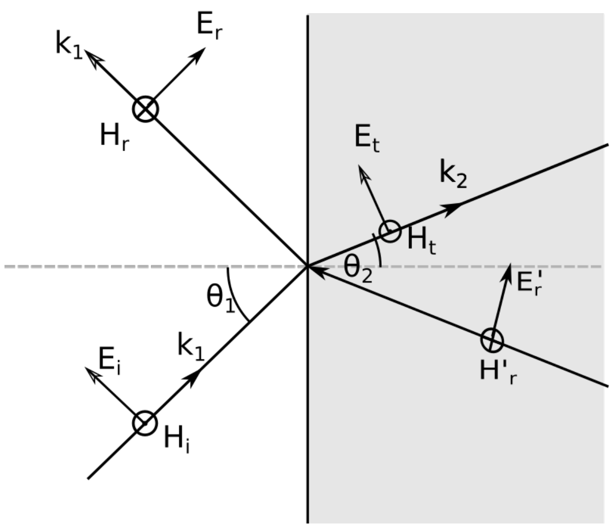


Figure 2.3: Reflection and transmission for P waves. The primed amplitudes are for an artificial wave which provides symmetry to the boundary condition equations and facilitates the introduction of the matrix method.

In this way, Eq. 2.48 and Eq. 2.49 now can be written as:

$$\mathbb{E}_i \cos\theta_i + \mathbb{E}_r \cos\theta_i = \mathbb{E}_t \cos\theta_t + \mathbb{E}_r' \cos\theta_t \tag{2.54}$$

$$\sqrt{\frac{\epsilon_1}{\mu_1}}[\mathbb{E}_i - \mathbb{E}_r] = \sqrt{\frac{\epsilon_2}{\mu_2}}[\mathbb{E}_t - \mathbb{E}_r'] \tag{2.55}$$

This two equations can be rewritten in matrix form as follows:

$$\begin{pmatrix} E_i \\ E_r \end{pmatrix} \begin{pmatrix} \cos\theta_i & \cos\theta_i \\ \sqrt{\frac{\epsilon_1}{\mu_1}} & -\sqrt{\frac{\epsilon_1}{\mu_1}} \end{pmatrix} = \begin{pmatrix} E_t \\ E_r' \end{pmatrix} \begin{pmatrix} \cos\theta_t & \cos\theta_t \\ \sqrt{\frac{\epsilon_2}{\mu_2}} & -\sqrt{\frac{\epsilon_2}{\mu_2}} \end{pmatrix} \tag{2.56}$$

Here we identify the Dynamical Matrix $D_P(i)$ for P waves as

$$D_P(i) = \begin{pmatrix} \cos\theta_i & \cos\theta_i \\ \sqrt{\frac{\epsilon_i}{\mu_i}} & -\sqrt{\frac{\epsilon_i}{\mu_i}}, \end{pmatrix} \tag{2.57}$$

or in terms of the refraction index as

$$D_P(i) = \begin{pmatrix} \cos\theta_i & \cos\theta_i \\ n_i & -n_i, \end{pmatrix} \tag{2.58}$$

where i refers to the medium and the angle.

On the other hand, following the same geometry that for the P wave, we obtain the boundary conditions for S waves as:

$$\mathbb{E}_i + \mathbb{E}_r = \mathbb{E}_t \tag{2.59}$$

$$\sqrt{\frac{\epsilon_1}{\mu_1}}[\mathbb{E}_i \cos\theta_i - \mathbb{E}_r \cos\theta_i] = \sqrt{\frac{\epsilon_2}{\mu_2}}\mathbb{E}_t \cos\theta_t \tag{2.60}$$

This two equations can be rewritten in matrix form as:

$$\begin{pmatrix} E_i \\ E_r \end{pmatrix} \begin{pmatrix} 1 & 1 \\ \sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_i & -\sqrt{\frac{\epsilon_1}{\mu_1}}\cos\theta_i \end{pmatrix} = \begin{pmatrix} E_t \\ E_r' \end{pmatrix} \begin{pmatrix} 1 & 1 \\ \sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta_t & -\sqrt{\frac{\epsilon_2}{\mu_2}}\cos\theta_t \end{pmatrix} \tag{2.61}$$

Here we identify the Dynamical Matrix $D_S(i)$ for S waves as

$$D_S(i) = \begin{pmatrix} 1 & 1 \\ \sqrt{\frac{\epsilon_i}{\mu_i}}\cos\theta_i & -\sqrt{\frac{\epsilon_i}{\mu_i}}\cos\theta_i \end{pmatrix} \tag{2.62}$$

In terms of the reflection index:

$$D_S(i) = \begin{pmatrix} 1 & 1 \\ n_i\cos\theta_i & -n_i\cos\theta_i \end{pmatrix} \tag{2.63}$$

2.3 Reflectance and Transmittance

The Fresnel equations give the ratios of the amplitudes of the reflected wave and the transmitted wave to the amplitude of the incident wave. To find how much energy is reflected from the boundary and transmitted into the second media, we need to consider the ratios of the Poynting power flow of the reflected and the transmitted waves to that of the incident wave. The power flow parallel to the boundary surface is unaffected and is a constant throughout the medium. [7]

The reflectance is defined in general by:

$$R = \frac{I_r}{I_i}, \tag{2.64}$$

where I_r is the reflected intensity defined by $I =< S^{\perp} >$, where $S^{\perp} = \widehat{x} \cdot \mathbb{S}$ is the perpendicular to the interface component of $\mathbb{S}$, the Poynting's vector of the incident wave, and the rectangular brackets denote the time-average, which for a given function $f(t)$ is defined as $< f(t) >= \frac{1}{T} \int_0^T f(t)dt$. The Poynting's vector $\mathbb{S}$ is given by Eq. 2.49, but here we will use the expression for linear media that is given by Eq. 2.50:

$$\mathbb{S} = \frac{\mathbb{E} \times \mathbb{B}}{\mu} \tag{2.65}$$

$$\mathbb{S} = \mathbb{E} \times \mathbb{H} \tag{2.66}$$

Where $\mathbb{E} = \mathbb{E}_0(\cos \omega t)$ and $\mathbb{E} = \mathbb{H}_0(\cos \omega t)$, then we get:

$$S = \mathbb{E} \times \mathbb{H} = \mathbb{E}_0 \times \mathbb{H}_0 \cos^2(\omega t) \tag{2.67}$$

$$S^{\perp} = S \cdot \widehat{x} = \widehat{x} \cdot (\mathbb{E}_0 \times \mathbb{H}_0) \cos^2(\omega t) \tag{2.68}$$

$$< S^{\perp} > = < \widehat{x} \cdot (\mathbb{E}_0 \times \mathbb{H}_0) \cos^2(\omega t) > \tag{2.69}$$

The only part depending on time is $\cos^2(\omega t)$, so we have

$$< S^{\perp} >= \widehat{x} \cdot (\mathbb{E}_0 \times \mathbb{H}_0) < \cos^2(\omega t) >, \tag{2.70}$$

then,

$$< S^{\perp} > = \widehat{x} \cdot (\mathbb{E}_0 \times \mathbb{H}_0) \frac{1}{\pi} \int_0^{\pi} cos^2(\omega t)dt \tag{2.71}$$

$$< S^{\perp} > = \widehat{x} \cdot (\mathbb{E}_0 \times \mathbb{H}_0) \frac{1}{\pi} \frac{1}{2} [(wt) + \sin(wt)\cos(wt)]|_0^{\pi} \tag{2.72}$$

$$< S^{\perp} > = \widehat{x} \cdot (\mathbb{E}_0 \times \mathbb{H}_0) \frac{1}{\pi} \frac{\pi}{2} \tag{2.73}$$

$$< S^{\perp} > = \frac{1}{2} \widehat{x} \cdot (\mathbb{E}_0 \times \mathbb{H}_0) \tag{2.74}$$

If $\mathbb{H} = \frac{\mathbb{B}_0}{\mu} = \frac{\widehat{k} \times \mathbb{E}_0}{\mu \omega}$, thus,

$$I = \frac{1}{2} \widehat{x} \cdot \frac{k \mathbb{E}_0^2}{\mu \omega} \widehat{k} \tag{2.75}$$

$$I = \frac{k_x E_0^2}{2 \mu \omega}. \tag{2.76}$$

So we have that I_r and I_i are respectively

$$I_r = \frac{k_1 E_{0r}^2 \cos \theta_i}{2 \mu_1 \omega} \tag{2.77}$$

$$I_i = \frac{k_1 E_{0i}^2 \cos \theta_i}{2 \mu_1 \omega} \tag{2.78}$$

Replacing Eq. 2.61 and Eq. 2.62 in the equation Eq. 2.48 to obtain the reflectance

$$R = \frac{\frac{k_1 E_{0r}^2 \cos\theta_i}{2\mu_1\omega}}{\frac{k_1 E_{0i}^2 \cos\theta_i}{2\mu_1\omega}} \tag{2.79}$$

$$R = \left|\frac{E_{0r}^2}{E_{0i}^2}\right| \tag{2.80}$$

$$R_{s,p} = |r_{s,p}|^2 \tag{2.81}$$

On the other hand the Transmittance is given by

$$T = \frac{I_t}{I_i}, \tag{2.82}$$

where,

$$I_t = \frac{k_2 E_{0t}^2 \cos\theta_t}{2\mu_1\omega} \tag{2.83}$$

$$I_i = \frac{k_1 E_{0i}^2 \cos\theta_i}{2\mu_1\omega} \tag{2.84}$$

Replacing Eq. 2.67 and Eq. 2.68 in Eq. 2.66,

$$T = \frac{\frac{k_2 E_{0t}^2 \cos\theta_t}{2\mu_1\omega}}{\frac{k_1 E_{0i}^2 \cos\theta_i}{2\mu_1\omega}} \tag{2.85}$$

With $\mu_2 \approx \mu_1$,

$$T = \frac{k_2}{k_1}\frac{\cos\theta_t}{\cos\theta_i}\left|\frac{E_{0t}^2}{E_{0i}^2}\right| \tag{2.86}$$

$$T_{s,p} = \frac{k_2}{k_1}\frac{\cos\theta_t}{\cos\theta_i}|t_{s,p}|^2 \tag{2.87}$$

So, we have obtained the reflectance and the transmittance in terms of the Fresnel coefficients r and t for S and P waves in the equations Eq. 2.65 and Eq. 2.71.

As an example of this calculations, for an interface between semi-infinities dielectric media Air and BK7, with refractive index 1 and 1.51, respectively, we have obtained the reflectance and the transmittance for P waves as function of the incident angles (Fig. 2.4). Here we identify the well known Brewster's angle at which the reflectance goes to zero.

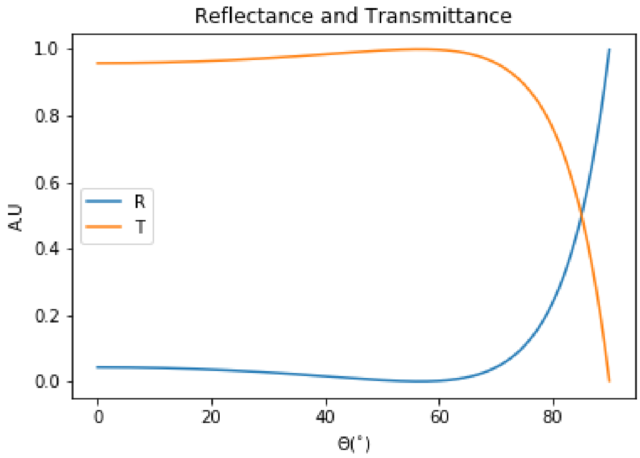


Figure 2.4: Reflectance and Transmittance for P waves in a single interface Air-BK7.

Chapter 3

Single Homogeneous and Isotropic Layer

In this chapter, we treat the optics of the simplest layered structure consisting on a finite layer between two semi-infinite media. We shall present specifically how to calculate the reflectance and transmittance as function of the parameters of the incident wave and those of the layered systems as number of layers, materials, and thicknesses.

The plane of incidence is chosen to be the x-y plane, and the interfaces connecting the different layers are chosen to be parallel to the y-z plane. In this geometry, the electric field of the waves can be written as

$$\mathbb{E}(\mathbf{r}, t) = \mathbb{E}(x) \exp^{i(wt - \beta z)}, \tag{3.1}$$

where β is the so called propagation constant defined as

$$\beta = (n_0 \omega / c) \sin \theta_0, \tag{3.2}$$

being n_0 and θ_0 the refractive index and the propagation angle of in the incident media, respectively. This is because, according to Snell's law, the z component of the propagation vector is constant, while the discontinuities in the structure are along the x axis.[7] Therefore, in what follows, we only consider the x-component in the spatial phase of the propagation wave.

3.1 2 x 2 matrix method

Let us consider a mono-layer of refractive index n_1 between two semi-infinite media: the so called incident medium of refractive index n_0 and the substrate of refractive index n_2, in the geometry shown in Fig. (3.1).

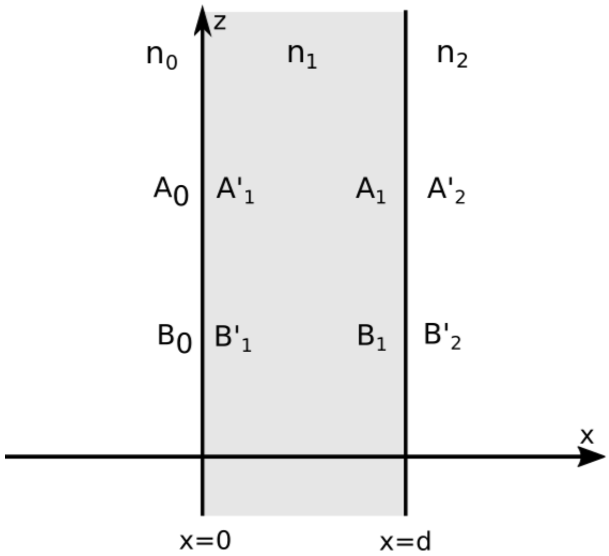


Figure 3.1: Geometry of a mono-layer treat with the matrix method

An incoming wave travels from the left side. The transmitted wave to the layer suffers many reflections in each of the boundary interfaces giving rise to a superposition of traveling waves to the right side and a superposition of traveling waves to the left side. On the other hand, a collection of waves emerges from the layer back to the incident media, which we call reflected wave, and another set of waves emerges to the substrate, which we call the transmitted wave. Therefore, we are concerned with six waves: three traveling to the right and labeled with amplitudes A, and three going back to the left and labeled with amplitudes B. Although the wave with amplitude B_2 does not have physical meaning (there is not an incoming wave from the right side of the substrate since it is considered to be infinity), we keep this artificial wave just to give symmetry to the boundary condition equations. We will later drop this amplitude to zero to recover the physical meaning in the solution of the problem. The unprimed amplitudes represent the amplitudes of the waves just at the left side of the interfaces, while the primed are used to identify the amplitudes of the wave just at the right side of the interfaces.

We recall for the reader the boundary conditions given in section 2.2

$$\begin{pmatrix} E_i \\ E_r \end{pmatrix} D_1 = \begin{pmatrix} E_t \\ E'_r \end{pmatrix} D_2, \tag{3.3}$$

where D_1 and D_2 are the Dynamical matrices in each medium for the respective polarization of the waves. For the actual problem Eq. 3.3 reads

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} D_0 = D_1 \begin{pmatrix} A'_1 \\ B'_1 \end{pmatrix} \tag{3.4}$$

Where the amplitudes of the medium 0, can be expressed in terms of the dynamical matrices and the amplitudes of the medium 1 reversing D_0:

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = D_0^{-1} D_1 \begin{pmatrix} A'_1 \\ B'_1 \end{pmatrix} \tag{3.5}$$

Having examined what is happening in the first boundary, we have to know now what happen in the layer, for that we can do something similar:

$$\begin{pmatrix} A_1' \\ B_1' \end{pmatrix} = P_1 \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} \tag{3.6}$$

Where P_1 is the Matrix Propagation for the medium 1 given by:

$$P_1 = \begin{pmatrix} e^{i\phi_1} & 0 \\ 0 & e^{-i\phi_1} \end{pmatrix} \tag{3.7}$$

where $\phi = k_{1x} \cdot d$ which depends of the thickness of this layer (d) and the x component of the wave vector (k_{1x}).

Finally, we analyze the last interface of the system and get:

$$\begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = D_1^{-1} D_2 \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} \tag{3.8}$$

Then, the amplitudes A_0, B_0, A_2' and B_2' are related by

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = D_0^{-1} D_1 P_1 D_1^{-1} D_2 \begin{pmatrix} A_2' \\ B_2' \end{pmatrix} \tag{3.9}$$

Knowing that[1]:

$$D_i^{-1} D_j = D_{ij} = \frac{1}{t_{12}} \begin{pmatrix} 1 & r_{12} \\ r_{12} & 1 \end{pmatrix} \tag{3.10}$$

The Eq. 3.9 can be written finally as:

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = D_{01} P_1 D_{12} \begin{pmatrix} A_2' \\ B_2' \end{pmatrix} \tag{3.11}$$

3.2 Alternative Derivation

Referring to the structure and geometry shown in the Fig. (3.2)

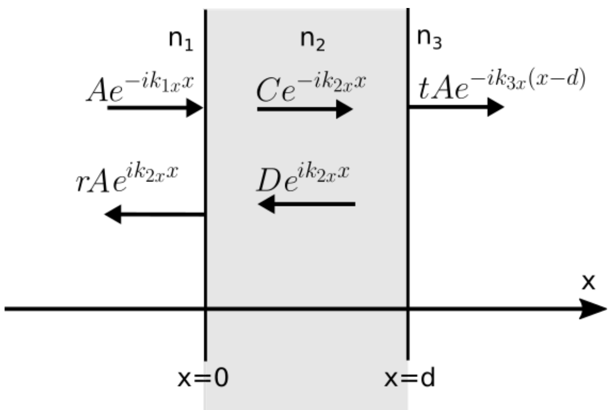


Figure 3.2: Alternative Derivation

we can write the relation between the waves in right and left from the interface $x = 0$

[1]For the detailed procedure see Appendix A.1

$$
\begin{aligned}
C &= t_{12}A + r_{21}D && (3.12)\\
rA &= r_{12} + t_{21}D && (3.13)
\end{aligned}
$$

On the other interface ($x = d$) based on the definition of the formulas for reflection and transmission coefficients, we can write the relation between these waves as:

$$
\begin{aligned}
tA &= t_{23}Ce^{-ik_{2x}d} && (3.14)\\
De^{ik_{2x}d} &= r_{23}Ce^{-ik_{2x}d} && (3.15)
\end{aligned}
$$

Where $t_{12}, t_{21}, r_{12}, r_{21}, t_{23}, r_{23}$ are the transmission and reflection coefficients, respectively associated with each interface.

We have a system of four equations from which we can find the transmission and reflection coefficients of the system t and r.

Using the Eq. 3.3 and Eq. 3.6 to eliminate A, C and D we find that:

$$
\begin{aligned}
r &= r_{12} + \frac{t_{12}t_{21}r_{23}e^{-ik_{2x}d}}{1 - r_{21}r_{23}e^{-ik_{2x}d}} && (3.16)\\
t &= \frac{t_{12}t_{23}e^{-ik_{2x}d}}{1 - r_{21}r_{23}e^{-ik_{2x}d}} && (3.17)
\end{aligned}
$$

Also we can obtain the amplitude of the waves contained in the single layer C and D in terms of the amplitude of the incident wave which we define.

$$
\begin{aligned}
C &= \frac{t_{12}}{1 - r_{21}r_{23}e^{-ik_{2x}d}}A && (3.18)\\
D &= \frac{t_{12}r_{23}e^{-ik_{2x}d}}{1 - r_{21}r_{23}e^{-ik_{2x}d}}A && (3.19)
\end{aligned}
$$

If $\phi = k_{2x}d$, the equations from Eq. 3.12 to Eq. 3.15 are given now by

$$
\begin{aligned}
r &= r_{12} + \frac{t_{12}t_{21}r_{23}e^{-i\phi}}{1 - r_{21}r_{23}e^{-i\phi}} && (3.20)\\
t &= \frac{t_{12}t_{23}e^{-i\phi}}{1 - r_{21}r_{23}e^{-i\phi}} && (3.21)\\
C &= \frac{t_{12}}{1 - r_{21}r_{23}e^{-i\phi}}A && (3.22)\\
D &= \frac{t_{12}r_{23}e^{-i\phi}}{1 - r_{21}r_{23}e^{-i\phi}}A, && (3.23)
\end{aligned}
$$

from here, we get the amplitude of the waves in the layer, and therefore we have that in this case the amplitude of the electric field E is given by:

$$
E = Re[Ce^{-ik_x} + De^{ik_x}] \qquad (3.24)
$$

3.3 Examples using the computing code - Python

In this section we present, as an example, the reflectance and transmittance as a function of the angle of incidence, and the distribution of the field trough a monolayer (electric field profile) as a function of the transverse coordinate (x). For this, we wrote a computing code in Python (see Chap. 5).

The configuration considered consists of a gold monolayer of thickness $d = 38$ nm sandwiched between two semi-infinity media of BK7 and Air. The incoming wave is P-polarized with wavelength of $\lambda = 533$ nm. For the electric field profile we consider the case of an incident angle $\theta = 47°$,

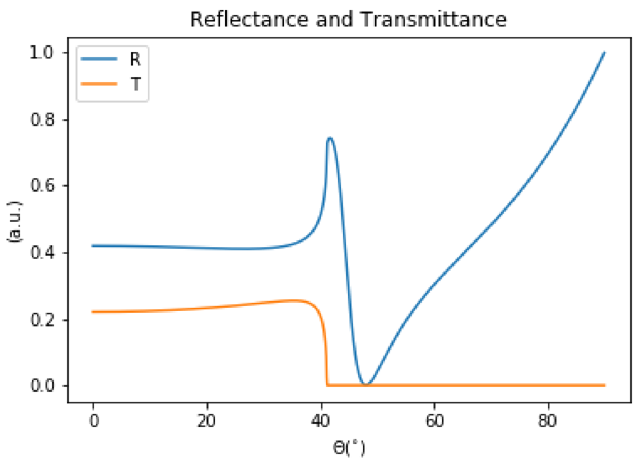


Figure 3.3: Reflectance and Transmittance for P polarization in gold layer, $d = 38$ nm, $\lambda = 533$ nm.

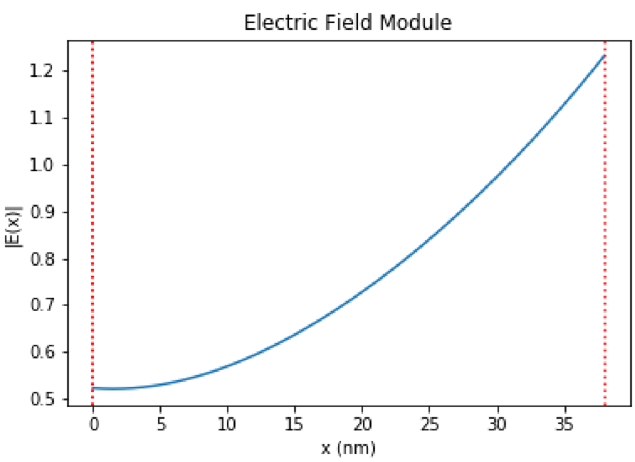


Figure 3.4: Module of the electric field for P polarization in gold layer, $d = 38$ nm, $\lambda = 533$ nm and $\theta = 47°$

now for the same system but for S polarization we have the following result:

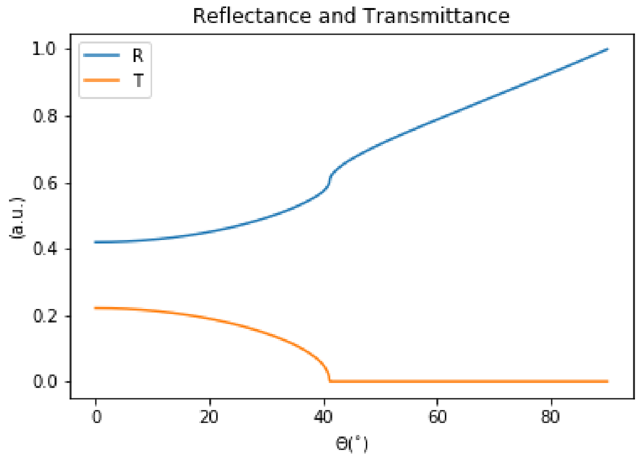


Figure 3.5: Reflectance and Transmittance for S polarization in gold layer, $d = 38$ nm, $\lambda = 533$ nm.

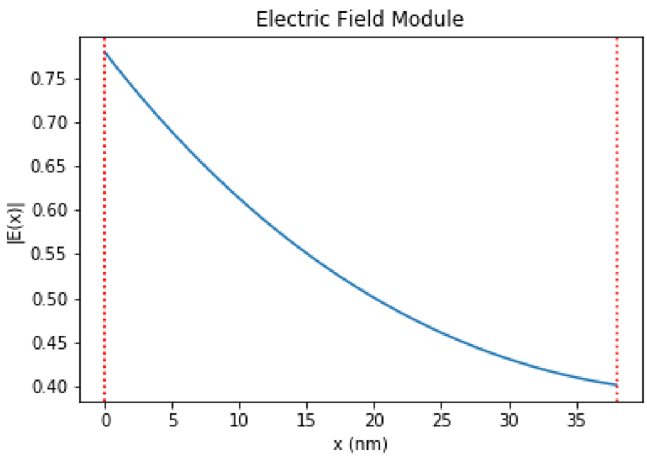


Figure 3.6: Module of the electric field for S polarization in gold layer, $d = 38$ nm, $\lambda = 533$ nm and $\theta = 47°$

Chapter 4

Optics of a Multilayer and Homogeneous System

In this chapter we will use the Matrix formulation to solve the amplitudes of electric field of any isotropic layered media. When the number of layers is too large, the analysis is very complicated because of the number of equations involved. For that reason, we will use a new matrix method that is a systematic approach for problems of this kind.

4.1 Transfer Matrix Method (Multilayer system)

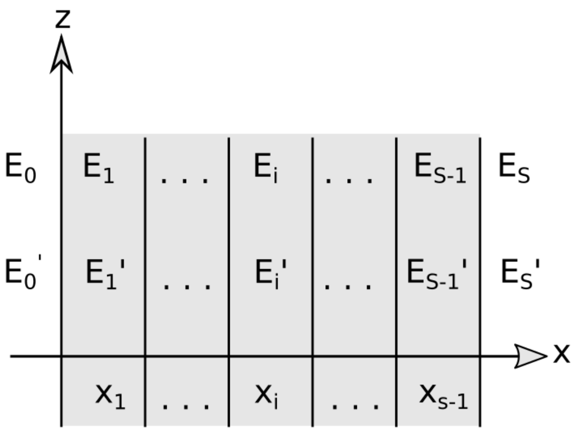


Figure 4.1: Multilayer Structure

For this structure we want to find an expression that let us determine the amplitudes of the waves in each layer as well as the Fresnel's coefficients for all structure based on the amplitude of the incident wave. For that we will use the same method that we used in the previous section for a single layer, which can be generalized for any multilayer system.

Referring to our system in Fig. 4.1, E_i and E_i' are the amplitudes of the right and left travelling waves in each layer. In the first medium with refractive index (n_0) there are two waves: E_0 is the amplitude of the incident wave which we determine, E_0' is the amplitude of the reflected wave from the system, and E_s is the amplitude of the transmitted wave to the substrate.

Following the same sense of Eq. 3.2, the general expression that links the amplitudes E_0, E_0', E_s and E_s' is given by:

$$\begin{pmatrix} E_0 \\ E_0' \end{pmatrix} = D_0^{-1} \prod_{l=1}^{s-1} (D_l P_l D_l^{-1}) D_s \begin{pmatrix} E_s \\ E_s' \end{pmatrix} \tag{4.1}$$

Where $D_0^{-1} \prod_{l=1}^{s-1} (D_l P_l D_l^{-1}) D_s$ is called the Transfer Matrix of the system (TM_S), which result is always given by this form:

$$TM_S = \begin{pmatrix} M_{00} & M_{01} \\ M_{10} & M_{11} \end{pmatrix} \tag{4.2}$$

So, the equation Eq. 4.2 is rewritten as:

$$\begin{pmatrix} E_0 \\ E_0' \end{pmatrix} = \begin{pmatrix} M_{00} & M_{01} \\ M_{10} & M_{11} \end{pmatrix} \begin{pmatrix} E_s \\ E_s' \end{pmatrix} \tag{4.3}$$

After solving the right side of Eq. 4.1 we obtain:

$$\begin{pmatrix} E_0 \\ E_0' \end{pmatrix} = \begin{pmatrix} E_s M_{00} + E_s' M_{01} \\ E_s M_{10} + E_s' M_{11} \end{pmatrix} \tag{4.4}$$

Assuming the amplitude of the incident wave $E_0 = 1$, and $E_s' = 0$ because there is not an incoming wave from the substrate. Therefore E_s is the transmittance coefficient and E_0' is the reflectance coefficient of the system and are given by the next equations:

$$t = \frac{1}{M_{00}} \tag{4.5}$$

$$r = \frac{M_{10}}{M_{00}} \tag{4.6}$$

4.2 Distribution of the Electric Field in the structure (E profile)

Equation (4.1) describes the effective response of the system in terms of the reflected and transmitted waves related to the incident one, regardless the details of the electric field inside the structure. Although field distribution through the system cannot be measured experimentally, studying the field profile in the multilayer is of great interest to identify optical excitations in the system, unusual absorption and field spots. In this section we present the analytic calculation to find the electric field profile in any layer of the system.

Knowing that all that happen in each layer does not affect the effective response of the system, we use the transfer matrix method, in the first layer the expression to get the amplitudes of the waves is given by:

$$\begin{pmatrix} E_0 \\ E_0' \end{pmatrix} = D_0^{-1} D_1 \begin{pmatrix} E_1 \\ E_1' \end{pmatrix} \tag{4.7}$$

Where in general way:

$$D_\alpha^{-1} D_\beta = D_{\alpha\beta} \tag{4.8}$$

Then, in the second and third layer:

$$\begin{pmatrix} E_0 \\ E_0' \end{pmatrix} = D_{01}P_1D_{12}\begin{pmatrix} E_1 \\ E_1' \end{pmatrix} \tag{4.9}$$

$$\begin{pmatrix} E_0 \\ E_0' \end{pmatrix} = D_{01}P_1D_{12}P_2D_{23}\begin{pmatrix} E_3 \\ E_3' \end{pmatrix} \tag{4.10}$$

Generalizing for the i-th layer:

$$\begin{pmatrix} E_0 \\ E_0' \end{pmatrix} = D_0^{-1}\prod_{l=1}^{i-1}(D_lP_lD_l^{-1})D_i\begin{pmatrix} E_i \\ E_i' \end{pmatrix} \tag{4.11}$$

Arbitrary defining the incident amplitude E_0 as unity, we obtain the amplitude of E_0'. The expression $D_0^{-1}\prod_{l=1}^{i-1}(D_lP_lD_l^{-1})D_i$ is called the Transfer Matrix up to the i-th layer (TM_i), which is given by

$$TM_i = \begin{pmatrix} N_{00} & N_{01} \\ N_{10} & N_{11}. \end{pmatrix} \tag{4.12}$$

Operating and simplifying we get that the amplitudes of the waves in the i-th layer are given by

$$E_i = A_0N_{00} + (N_{01}M_{10})/M_{00} \tag{4.13}$$

$$E_i' = A_0N_{10} + (N_{11}M_{10})/M_{00} \tag{4.14}$$

Where N_{00}, N_{00}, N_{01}, N_{10} and N_{11} are values of TM_i, M_{00} and M_{10} are values of TM_s and A_0 is the amplitude of the incident wave.

4.3 Examples

Here we present some examples of the electric field profile for different configurations. We start with the same configuration treated in the last section: mono-layer of gold ($d = 38nm$) in air substrate and BK7 as incident media (Kretschmann configuration), for P- and S-polarized waves.

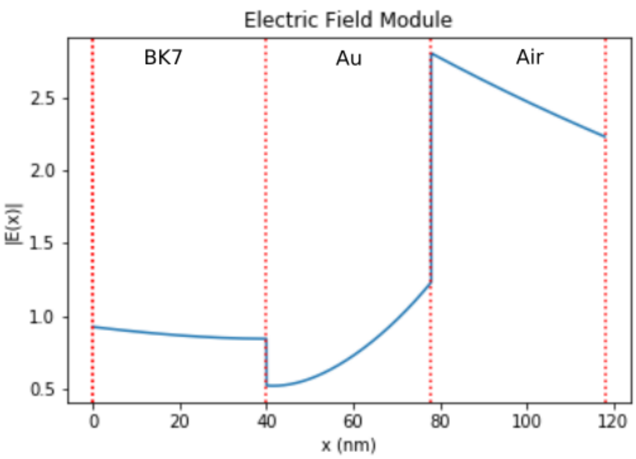


Figure 4.2: Module of the electric field for P-waves for a gold layer ($d = 38$ nm) between BK7 and Air (substrate).

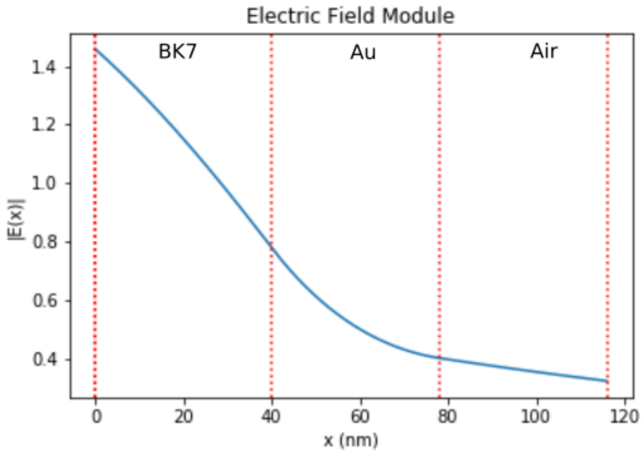


Figure 4.3: Module of the electric field for S-waves for a gold layer ($d = 38$ nm) between BK7 and Air (substrate).

Here we can see for, P-waves (electric field with a normal-to-the-interface component), the discontinuities of the field across the interfaces (Gauss law), while for S-waves (electric field parallel to the interface) the field is continuous (Faraday's law). On the other hand, we can see the usual enhancement of the electric field in this configuration for P-waves. This is related to the excitation of surface plasmons in the interface gold-air. The distinctive evanescent wave in this interface demonstrate the strong confinement of the electric field to the surface associated with such a optical excitation, meanwhile for S-polarized waves (unable to excite surface plasmons), we can see the usual absorption of the wave along the propagating direction.

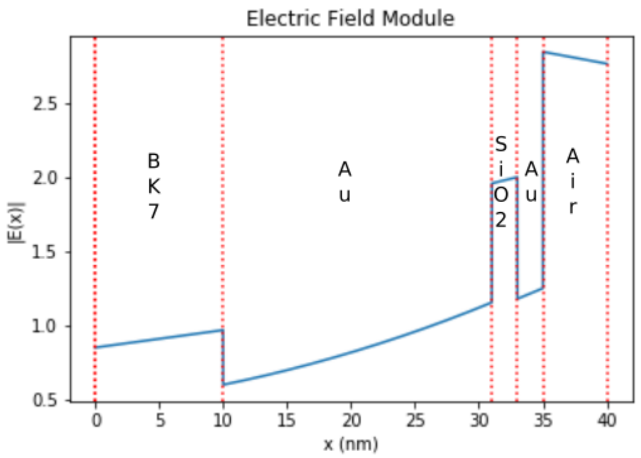
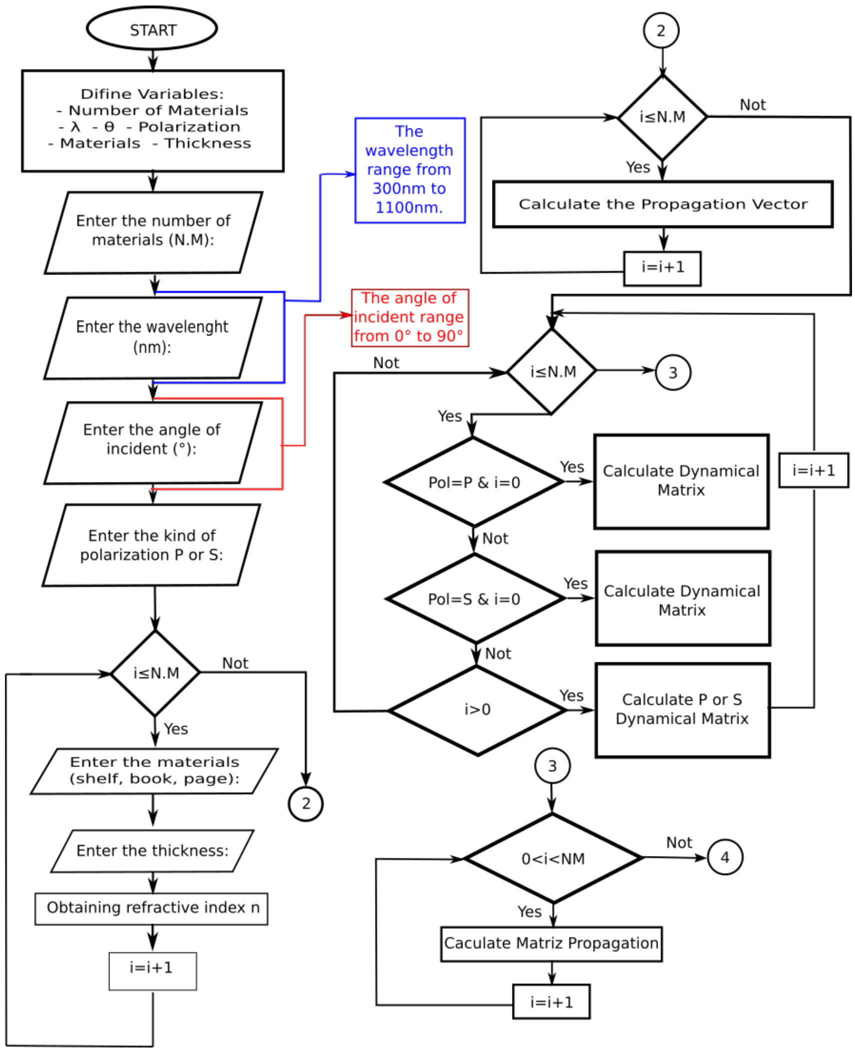


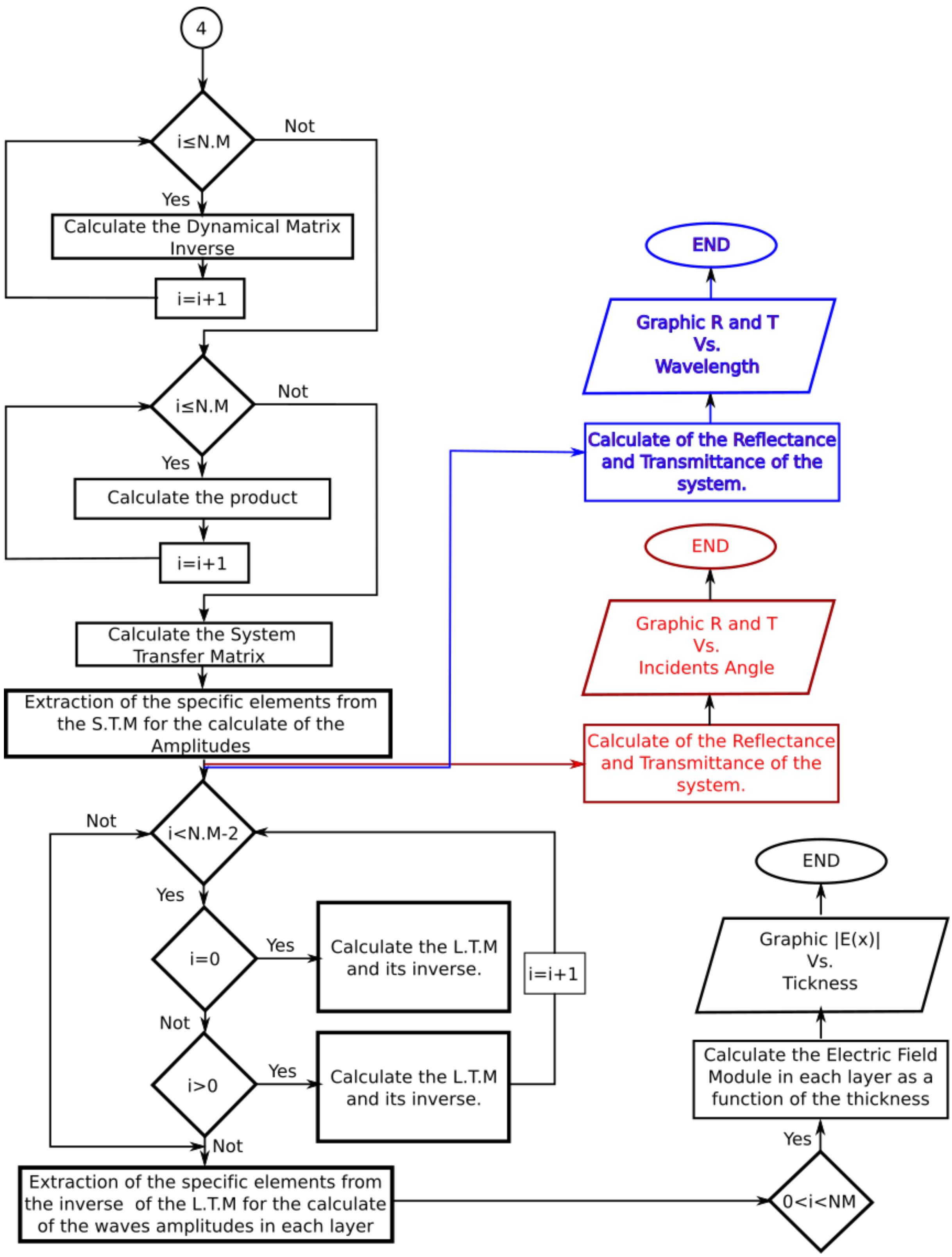
Figure 4.4: Module of the electric field for P-waves for configuration Gold ($d = 21$ nm), Silicon dioxide ($d = 2$ nm) and Gold ($d = 2$ nm) between BK7 and Air (substrate).

Chapter 5

Computational Method

In this chapter, we present the respective flowchart of the programming code for both programs created in this project. We show black text, that represent the flowchart of the program which plot the Electric Field Module, on the other hand the red text specify the differences in first program, to create other which plot the Reflectance and Transmittance.




4
i≤N.M
Not
Yes
Calculate the Dynamical Matrix Inverse
i=i+1
i≤N.M
Not
Yes
Calculate the product
i=i+1
Calculate the System Transfer Matrix
Extraction of the specific elements from the S.T.M for the calculate of the Amplitudes
i<N.M-2
Not
Yes
i=0
Yes
Calculate the L.T.M and its inverse.
Not
i>0
Yes
Calculate the L.T.M and its inverse.
i=i+1
Not
Extraction of the specific elements from the inverse of the L.T.M for the calculate of the waves amplitudes in each layer
END
Graphic R and T Vs. Wavelength
Calculate of the Reflectance and Transmittance of the system.
END
Graphic R and T Vs. Incidents Angle
Calculate of the Reflectance and Transmittance of the system.
END
Graphic |E(x)| Vs. Tickness
Calculate the Electric Field Module in each layer as a function of the thickness
Yes
0<i<NM

Chapter 6

Conclusions and Perspectives

- The 2x2 matrix formalism to solve the problem of electromagnetic waves propagating along a multilayer system was reviewed.
- The electric field profile along a the transverse coordinate of a multilayer system, made of linear, isotropic, and homogeneous materials, has been calculated using a computing program code write were written to compute the optical response functions of such systems. Namely, reflectance and transmittance as functions of the incident angle (subroutine 1), wavelength (subroutine 2), and linear polarization state. These two programs allows us for testing the results in the main program.
- This computational program is part of the products related to the research project of my group (GOMa) of the Universidad Distrital Francisco José de Caldas. Hence, it is expected to be a computational tool further research.

Appendix A

Product Of Dynamical Matrices

*

The Dynamical Matrix is given by the equations Eq. 2.58 y Eq. 2.63 for P and S wave respectively. Using this equations and the reflection and transmission coefficients for each case. We will show the procedure to deduce the definition given in the section 3.2 by the equation Eq. 4.8.

If we use this definition:

$$D12 = D_1^{-1} D_2 \tag{A.1}$$

A.1 P Waves

The matrix for the inverse of the D_1 and for the D_2 are given by:

$$D_1^{-1} = \begin{pmatrix} \frac{1}{2\cos\theta_i} & \frac{1}{2n_1} \\ \frac{1}{2\cos\theta_i} & -\frac{1}{2n_1} \end{pmatrix} \tag{A.2}$$

$$D_2 = \begin{pmatrix} \cos\theta_t & \cos\theta_t \\ n_2 & -n_2 \end{pmatrix} \tag{A.3}$$

The product of this matrices is:

$$D_1^{-1} D_2 = \begin{pmatrix} \frac{\cos\theta_t}{2\cos\theta_i + n_2/2n_1} & \frac{\cos\theta_t}{2\cos\theta_i - n_2/2n_1} \\ \frac{\cos\theta_t}{2\cos\theta_i - n_2/2n_1} & \frac{\cos\theta_t}{2\cos\theta_i + n_2/2n_1} \end{pmatrix} \tag{A.4}$$

$$D_1^{-1} D_2 = \begin{pmatrix} \frac{1}{2}\frac{\cos\theta_t}{\cos\theta_i + n_2/n_1} & \frac{1}{2}\frac{\cos\theta_t}{\cos\theta_i - n_2/n_1} \\ \frac{1}{2}\frac{\cos\theta_t}{\cos\theta_i - n_2/n_1} & -\frac{1}{2}\frac{\cos\theta_t}{\cos\theta_i + n_2/n_1} \end{pmatrix} \tag{A.5}$$

$$D_1^{-1} D_2 = \begin{pmatrix} \frac{1}{2}\frac{n_1\cos\theta_t + n_2\cos\theta_i}{n_1\cos\theta_i} & \frac{1}{2}\frac{n_1\cos\theta_t - n_2\cos\theta_i}{n_1\cos\theta_i} \\ \frac{1}{2}\frac{n_1\cos\theta_t - n_2\cos\theta_i}{n_1\cos\theta_i} & \frac{1}{2}\frac{n_1\cos\theta_t + n_2\cos\theta_i}{n_1\cos\theta_i} \end{pmatrix} \tag{A.6}$$

Simplifying we get:

$$D_1^{-1} D_2 = \frac{1}{2}\frac{n_1\cos\theta_t + n_2\cos\theta_i}{n_1\cos\theta_i} \begin{pmatrix} 1 & \frac{n_1\cos\theta_t - n_2\cos\theta_i}{n_1\cos\theta_t + n_2\cos\theta_i} \\ \frac{n_1\cos\theta_t - n_2\cos\theta_i}{n_1\cos\theta_t + n_2\cos\theta_i} & 1 \end{pmatrix} \tag{A.7}$$

Replacing for equations Eq. 2.32 and Eq. 2.33 the result is given by:

$$D_1^{-1}D_2 = \frac{1}{t_{12}}\begin{pmatrix} 1 & r_{12} \\ r_{12} & 1 \end{pmatrix} \tag{A.8}$$

A.2 S Waves

In this case the D_1^{-1} and D_2 ar given by this matrices:

$$D_1^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2n_1\cos\theta_i} \\ \frac{1}{2} & -\frac{1}{2n_1\cos\theta_i} \end{pmatrix} \tag{A.9}$$

$$D_2 = \begin{pmatrix} 1 & 1 \\ n_2\cos\theta_t & -n_2\cos\theta_t \end{pmatrix} \tag{A.10}$$

The product of this matrices is:

$$D_1^{-1}D_2 = \begin{pmatrix} \frac{n_1\cos\theta_i + n_2\cos\theta_t}{2n_1\cos\theta_i} & \frac{n_1\cos\theta_i - n_2\cos\theta_t}{2n_1\cos\theta_i} \\ \frac{n_1\cos\theta_i - n_2\cos\theta_t}{2n_1\cos\theta_i} & \frac{n_1\cos\theta_i + n_2\cos\theta_t}{2n_1\cos\theta_i} \end{pmatrix} \tag{A.11}$$

Simplifying and replacing the equations Eq. 2.46 and Eq. 2.47 in Eq. $A.11$ we get:

$$D_1^{-1}D_2 = \frac{1}{t_{12}}\begin{pmatrix} 1 & r_{12} \\ r_{12} & 1 \end{pmatrix} \tag{A.12}$$

List of Figures

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